# **6. The Study of Individual Change** EPSY 8268

Individual change can be modeled across repeated measures *Yti*, based on the observed status on Y at time *t* for individual *i*, as a function of a systematic growth curve plus random error. Typically curves can be represented as a polynomial of degree *P*.

*Yti* = π0*i* + π1*iati* + π2*ia*2*ti* + … + *πPiaPti* + *eti*

The outcome score *Yti* is measured at time *t* for student *i* (for *i* from 1 to *n*), and *ati* is the age at time *t* for person *i* and *πPi* is the growth trajectory parameter *p* with the polynomial degree *P* (for *p* = 0, …, *P*). Carefully consider the centering of *a* in order to provide clear interpretation of π0*i*.

In some instances, a simple error structure can be assumed for *eti* , where *eti* ~ N(0, σ2). In other situations, a more complex error structure must be employed.

The level-2 model is at the level of the student *i*.

$$π\_{pi}=β\_{p0}+\sum\_{q=1}^{Qp}β\_{pq}X\_{qi}+r\_{pi}$$

 where *rpi*~ N(0, τ*pp*).

***Linear Growth***

Consider a linear growth model. When the number of time points is few or the time period is short, polynomial models are difficult to estimate. This is an unconditional model.

*Yti* = π0*i* + π1*i*(Time)*ti* + *eti* where *eti* ~ N(0, σ2).

π0*i* = β00 + β01(*X*)*i* + *r*0*i* where *r*0*i*~ N(0, τ00).

π1*i* = β10 + β11(*X*)*i* + *r*1*i* where *r*1*i*~ N(0, τ11), also with covariance, τ01.

π0*i*

π1*i*

*eti*

β00

β01(*X*)*i*

*r*0*i*

β10

β11(*X*)*i*

*r*1*i*

A couple notes on the model.

1. The model includes two levels over time (time nested within person). This can be generalized to three or four (or more) levels, by considering persons nested within organizations and so on. In some cases you will not have all of these levels. During the first two time points, you will not have the *Time* level. You can use the Prior-Year covariate model after the second time point, within person, and then move right to the group model at level 2.
2. Consider centering Time so that Time-1 is 0, Time-2 is 1, Time-3 is 2, etc. This will make the intercept the score at Time-1 (the intercept is the value of the outcome when the predictors are 0). At all other levels, except the final level, you should group-mean center the explanatory variables. By group-mean centering the variables, for example person variables, the intercept for each model (each level with predictors) will be the mean at that level.

Consider an example with time-points nested in students nested in teachers nested in schools. If you group-mean center student variables at level 2, the intercept at this level is the student baseline (beginning score if Time-1 = 0). At level 3, if you group-mean center the teacher variables, the intercept is the teacher baseline. At level 4, if you grand-mean center the school variables, the intercept is the grand mean across schools

1. When you group-mean center variables at one level, you should add to the model of the intercept at the next level the level-means to account for group differences. Group-mean centering takes out the mean differences because they are subtracted from each case in the group – so that each group has a mean of zero. We typically add these back into the model of the intercept at the next level, to account for group differences.

For example, a student’s prior score may have an effect on their current score, but so may the average score of the classroom where that student is located. At level 1, we add the group-mean centered prior score to account for student differences in prior achievement. Then at level 2, to model the intercept, we add the classroom average score to account for differences in classrooms. This fully accounts for the effect of that variable, including the within-group effect (βw at level 1) and the between group effect (βb at level 2). This also allows for the estimation of composition effects.

1. Finally, there are many coefficients estimated in growth models, particularly multi-level growth models, for which you may not be interested. The coefficients of most interest tend to include the grand-mean effects of student, classroom, and school variables on growth. In most cases, the grand mean intercept is not particularly interesting.

***Other Considerations***

*Variation in growth trajectories*

From a simple linear model, we can estimate the mean intercept, $\hat{β}\_{00}$, and mean growth rate, $\hat{β}\_{10}$. These are estimated with variance components (variance in π0*i* and π1*i* across individuals), which can be used to estimate standard deviations in individual growth trajectories.

We can estimate ranges of intercepts (perhaps baseline if time-point 1 = 0) with

$$\hat{β}\_{00}\pm 1.96\sqrt{τ\_{00}}$$

and growth rates using the standard deviations:

$$\hat{β}\_{10}\pm 1.96\sqrt{τ\_{11}}$$

*Reliability of growth trajectories*

$$reliability of \hat{π}\_{pi}=\frac{Var(π\_{pi})}{Var(\hat{π}\_{pi})}=\frac{τ\_{pp}}{(τ\_{pp}+v\_{ppi})}$$

*Correlation of change with initial status*

$$\hat{ρ}\left(π\_{0i},π\_{1i}\right)=\frac{\hat{τ}\_{01}}{\sqrt{\hat{τ}\_{00}\hat{τ}\_{11}}}$$

Source: Raudenbush, S.W. & Bryk*,* A.S. (2002). *Hierarchical Linear Models. Applications and Data Analysis Methods* (2nd ed., pp. 160-203). Sage Publications.