# 8. Three and Four Level Models EPSY 8268

**Three-Level Models**

We can conceptualize three level models. Consider students within classrooms within schools:

 *i* = 1, 2, …, *njk* children within classroom *j* in school *k*,

 *j* = 1, 2, …, *Jk* classrooms within school *k*,

 *k* = 1, 2, …, *K* schools.

The unconditional model is:

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| Student-level model | Variance within level |
|  |  |
| *Yijk* = π0*jk* + *eijk* where *eijk* ~ N(0, σ2) | σ2 /(σ2 + τπ + τβ) |
|  |  |
| Classroom-level model |  |
|  |  |
| π0*jk* = β00*k* + *r*0*jk* where *r*0*jk*~ N(0, τπ) | τπ /(σ2 + τπ + τβ) |
|  |  |
| School-level model |  |
|  |  |
| β00*k* = γ000 + *u*00*k* where *u*00*k*~ N(0, τβ) | τβ /(σ2 + τπ + τβ) |

Reliability of = τπ /(τπ + σ2/*njk*) and reliability of

Generalized models:

*Yijk* = π0*jk* + π1*jk* *a1ijk* + π2*jk* *a2ijk* + … + *πPjk* *aPijk* + *eijk* with *P* student characteristics [*a*]

 where *epjkl* ~ *MN*(0, σ2).

π*pjk* = β*p*0*k* + with *Qp* predictors [*X*] in the *P* + 1 level-2 equations

 p is the associated level-1 predictor

q is the level-2 predictor

 where *rpjk* ~ *MN*(0, **T**π)

β*pqk* = γ*pq*0 + with *Spq* predictors in the *Qp* + 1 level-3 equations

 s is the level-3 predictor

 where *upqk* ~ *MN*(0, **T**β).

**Four-Level Models**

There are *i* = 1, …, *njkl* level-1 units (e.g., individuals), nested within each of *j* = 1, …, *Jkl* level-2 units (e.g., classrooms), nested within each of *k* = 1, …, *Kl* level-3 units (e.g., universities), nested within each of *l* = 1, …, *L* level-4 units (e.g., country).

Generalized models:

*Yijkl* = π0*jkl* + π1*jkl* *a1ijkl* + π2*jkl* *a2ijkl* + … + *πPjkl* *aPijkl* + *eijkl* with *P* individual characteristics [*a*]

 where *epjkl* ~ *MN*(0, σ2),

π*pjkl* = with *Qp* predictors [*X*] in the *P* + 1 level-2 equations

 p is the associated level-1 predictor

q is the associated level-2 predictor

 where *rpjkl* ~ *MN*(0, **T**π).

β*pqkl* = with *Spq* predictors in the *Qp* + 1 level-3 equations

 s is the associated level-3 predictor

 where *upqkl* ~ *MN*(0, **T**β).

*pqsl* = with *Gpqs* predictors in the *Spq* + 1 level-4 equations

 *g* is the associated level-4 predictor

 where *vpqsl* ~ *MN*(0, **T**γ).

For longitudinal data, we represent the first level as *t* = 1, …, *Tijk* time points. Interpretations below assume group-mean centering at levels 2 and 3, and grand-mean centering at level 4.

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| OVER TIME – Growth Model notation |
|  | Level-1 is time (*t*). |
| Level-1 *Ytijk* = π0*ijk* + π1*ijk*(*Time*)*tijk* + *etijk* | The achievement score *Ytijk* is measured at time *t* for student *i* in classroom *j* in school *k*. Time could be coded 0, 1, 2… to total number of years. |
| where *etijk* ~ N(0, σ2) |  |
|  | Level-2 is student (*i*). |
| Level-2 π0*ijk* = β00*jk* + β01*jk*(*Student Vars*)*ijk* + *r*0*ijk* | π0*ijk* is the first-year score (intercept) for student *i*. First year achievement levels can be modeled with *P* student variables. |
|  π1*ijk* = β10*jk* + β11*jk*(*Student Vars*)*ijk* + *r*1*ijk* | π1*ijk* is the achievement growth slope (growth per year) for student *i*. You can explain variation in student growth using student variables. |
| where *rpijk*~ *MN*(0, **T**π) |  |
|  | Level-3 is classroom (*j*). |
| Level-3: β00*jk* = γ000*k* + γ001*k*(*Teacher Vars*)*jk* + *u*00*jk* | β00*jk* is the average student first-year score for classroom *j*. You can explain variation with teacher or classroom variables. |
| β01*jk* = γ010*k* + *u*01*jk* | β01*jk* is the effect of student variables on first-year score in classroom *j*. |
| β10*jk* = γ100*k* + γ101*k*(*Teacher Vars*)*jk* + *u*10*jk* | β10*jk* is the average growth rate (slope) for students in classroom *j*. Variation in growth across classrooms can be explained by teacher or classroom variables. |
| β11*jk* = γ110*k* + *u*11*jk* | β11*jk* is the effect of kid variables on student growth rates (slopes) for classroom *j*.  |
| where *upqjk*~ *MN*(0, **T**β) |  |

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| --- | --- |
|  | Level-4 is school (*k*). |
| Level 4: γ000*k* =δ0000 + δ0001(*School Vars*)*k* + *v*000*k* | γ000*k* is the average first-year score for school *k*. This can be modeled using school variables. δ0000 is the grand mean first-year score (across all kids, classrooms, and schools). δ0001 is the effect of school variables on first-year score. |
| γ001*k* = δ0010 | γ001*k* is the effect of teacher variables on classroom first-year scores in school *k*. δ0010 is the grand mean effect of teacher variables on classroom first-year scores across schools. |
| γ010*k* = δ0100 | γ010*k* is the average effect of student variables in school *k* on first-year scores. δ0100 is the grand mean effect of student variables on first-year scores across schools. |
|  |
| γ100*k* = δ1000 + δ1001(*School Vars*)*k* + *v*100*k* | γ100*k* is the average growth rate (slope) for school *k*. This can be modeled using school variables. δ1000 is the grand mean growth rate. δ1001 is the effect of school variables on growth rate at the school level.  |
| γ101*k* = δ1010 | γ101*k* is the effect of teacher variables on classroom growth rate for school *k*. δ1010 is the grand mean effect of teacher variables on growth across schools. |
| γ110*k* = δ1100 | γ110*k* is the effect of student variables on student growth in school *k*. δ1100 is the grand mean effect of student variables on growth across schools. |
| where *vpqsk*~ N(0, **T**γ) |  |

Source: Raudenbush, S.W. & Bryk*,* A.S. (2002). *Hierarchical Linear Models. Applications and Data Analysis Methods* (2nd ed., pp. 160-203). Sage Publications.