**The Estimation and Role of Reliability in HLM** EPSY 8268

***A note on the use of reliability to estimate fixed-effects in HLM***

Level-1 parameters (intercepts and slopes of participants) are based on Bayesian shrunken estimates. They are empirical Bayes estimates of the true intercept and slope – which means that they are based on information from the group’s data in addition to some information from the data across all groups (the means). If the information from a group is not reliable, we supplement that information with more reliable information from the whole sample – thus, the Bayesian shrinkage idea – we shrink the estimates of the group toward the mean if the information from the group is not reliable.

To facilitate this process, each estimate (each intercept and slope) has an associated reliability. This reliability is a function of the ratio of parameter variance to observed variance (the proportion of variance that is in the parameters, or true in the classical test theory notion). The estimate for group *j* can be computed:

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The reliability λ*j* for the intercept, as an example, will be high when the intercepts for gropus, β0*j*, vary a great deal across groups or the sample size *nj* is large (the number of individuals within group). The same is true for slopes – reliability is high when groups vary in their slopes and the number of individuals within groups is large.

**Slope Precision and Reliability in HLM**

In HLM, slopes are estimated based on the regression of the outcome on the explanatory variable, within group. So each group is providing information to estimate a slope or magnitude of change in the outcome per unit change in the explanatory variable.

Each group will then have an intercept, and when the explanatory variable is centered in a meaningful way, the intercept can have a specific interpretation, such as the average. Each group will also have a slope, which estimates the change in outcome per unit change in the explanatory variable. Both the intercept and slope are randomly varying over groups, such that there is a grand mean intercept and slope, and potentially variances of intercepts and slopes across groups.

The group intercept and slope are group parameters. In addition, there are estimates of precision for each parameter, based on the amount of information available for estimating the intercept and slope and variability or stability in those estimates – the reliabilities in HLM.

Reliability of the intercept for group *j* is denoted as λ0*j*. This is the ratio of parameter variance to total variance (parameter + error variance). The traditional definition of reliability is the ratio of true-score variance to observed-score variance – the proportion of observed variance that is true variance. These variances are estimated in HLM so that parameter variance in the intercept is τ00 (between group variance) and total variance is the sum of τ00 and error variance for group *j*.

The basic Level-1 model is:

Score*ij* = β0*j* + β1*j*(*SESij* -$\overbar{X}\_{•j}$) + *rij*, where *rij* ~ N(0, $σ\_{e}^{2}$).

The basic Level-2 model is:

β0*j* = γ00 + *u*0*j*, where *u*0*j* ~ N(0, τ00), and

β1*j* = γ10 + *u*1*j*, where *u*1*j* ~ N(0, τ10).

Averaging across the *nj* observations within group *j* gives a model with the sample mean as the outcome: . This is a model where the sample group mean is an estimate of the group mean (intercept) β0*j*, and the error of estimation is , which has an error variance . *V*0*j* is the error variance, essentially a sampling error variance or the variance of  as an estimate of β0*j*. This is analogous to the error variance of estimating the mean – the SE(M) =$\sqrt{\frac{σ^{2}}{n}}$ or SE(M) = $\frac{s}{\sqrt{n}}$.

Now we can estimate the reliability of β0*j*, the intercept for group *j*:

 =  = .

In terms of classical test theory,  is a measure of the true parameter β0*j*. λ0*j* is the reliability because it is the ratio of true-score variance to total or observed-score variance, the extent to which what we observe is true (due to parameter variation). The reliability λ0*j* will be large when the group means, β0*j*, vary a great deal across groups or the *nj* number of individuals, is large.

Reliability λ1*j* of the slope β1*j* over persons is estimated similarly. For group *j*,



As with the reliability of the intercept, the reliability λ1*j* of the slope is large when group slopes vary a great deal across groups and the number of individuals within groups is large.

The average across groups provides a summary index of the reliability of the slope estimate for this population. The HLM program computes an average reliability for the estimates of the slope across the set of *j* level-2 groups.

Source: Raudenbush, S.W. & Bryk*,* A.S. (2002). *Hierarchical Linear Models. Applications and Data Analysis Methods* (2nd ed., pp. 46-50). Sage Publications.