

1. Thinking conceptually, describe the relationship between an eigenvector and the variables it is based on. _____
2. If \mathbf{A} is a square $p \times p$ matrix of full rank, $\text{Rank}(\mathbf{A}) =$ _____
3. If \mathbf{A} is a square $p \times p$ matrix of full rank, then \mathbf{A} has how many eigenvectors? _____

The following items refer to Principal Components Analysis and eigen analysis more generally.

4. Identify one typical use for principal components analysis. _____
5. If we conduct eigen analysis on a correlation matrix of 12 variables and the first two principal components result in $\lambda_1 = 3.5$ and $\lambda_2 = 2.5$, what proportion of variance has been explained by these two components? _____
6. If the following eigenvalues resulted from the principal components analysis of the 12 variable correlation matrix, how many components would you use to represent the space occupied by the original 12 variables? _____

$\lambda_1 = 3.5$	$\lambda_7 = 0.6$
$\lambda_2 = 2.5$	$\lambda_8 = 0.5$
$\lambda_3 = 1.3$	$\lambda_9 = 0.4$
$\lambda_4 = 1.1$	$\lambda_{10} = 0.3$
$\lambda_5 = 0.9$	$\lambda_{11} = 0.1$
$\lambda_6 = 0.7$	$\lambda_{12} = 0.1$
7. A matrix is positive definite if all of its eigenvalues are positive. What is the problem with having a negative eigenvalue? _____
8. Consider a case where you compute a new variable that is a linear combination of two other variables: the determinant of the correlation matrix becomes negative and the last eigenvalue is zero, so the number of eigenvalues (or eigenvectors) is $p - 1$. What does this indicate?
