Consider **A**, an n×n matrix, where the Rank of **A** = n.

- Full rank
- Nonsingular
- Linearly independent
- Determinant $\neq 0$
- Can invert **A**
- Can solve equations
- Unique solution

Normalizing

If $\underline{a}'\underline{b} = a_1b_1 + a_2b_2 + \ldots + a_nb_n = 0$, then $\underline{a}$ and $\underline{b}$ are orthogonal.

If $\underline{a}'\underline{a} = 1$, $\underline{a}$ is normalized.

We can normalize a vector: $\underline{x}^* = \frac{1}{\sqrt{\underline{x}'\underline{x}}}\underline{x}$.

$$\underline{a}'\underline{a} = 1 \rightarrow \left(\frac{1}{\sqrt{\underline{x}'\underline{x}}}\underline{x}\right)'\left(\frac{1}{\sqrt{\underline{x}'\underline{x}}}\underline{x}\right) = \left(\frac{1}{\sqrt{\underline{x}'\underline{x}}}\right)^2 \underline{x}'\underline{x} = \frac{1}{\underline{x}'\underline{x}}\underline{x}'\underline{x} = 1$$

Canonical Correlation

$$r = \frac{\text{cov}(x,y)}{\sqrt{\text{var}(x)}\sqrt{\text{var}(y)}} \text{ and } r^2 = \frac{\text{cov}(x,y)^2}{\text{var}(x)\,\text{var}(y)} = \text{cov}(x,y)^2\,\text{var}(x)^{-1}\,\text{var}(y)^{-1}.$$

$$\text{var}(y)^{-1}\,\text{cov}(x,y)\,\text{var}(x)^{-1}\,\text{cov}(x,y) = S_{yy}^{-1}S_{xy}S_{xx}^{-1}S_{xy}$$

Correlation between a single linear function of the Ys and a single linear function of the Xs.

Canonical correlation squared = λ, eigen value.

Row & Column Rank Equivalence

Consider an $n \times p$ matrix.

The n rows of the matrix are vectors in R^n, spanning the subspace called row space. The dimension of this space is called row rank.

The p columns of the matrix are vectors in R^p, spanning the subspace called column space. The dimension of this space is called column rank.

A theorem of matrix algebra proves row rank = column rank = rank.

Rank of an $n \times p$ matrix = rank of largest submatrix where $|\mathbf{A}| \neq 0$.

$\mathbf{A}$ is full rank if rank($\mathbf{A}$) = min{n,p}.